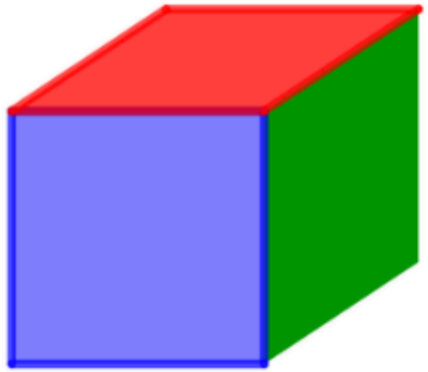
PROBLEM OF THE WEEK 1



Each face of a cube is to be painted. Six different colours of paint are available.

What is the smallest number of colours that are needed if faces that have an edge in common must not be painted the same colour?

Junior Mathematical Challenge, 1997.

United Kingdom Mathematics Trust

PROBLEM OF THE WEEK

Solution 1

3

Each pair of the three faces that come together at a vertex share an edge and therefore these faces must be painted different colours. So at least three colours are needed.

Opposite faces don't share an edge, so may be painted the same colour. There are three pairs of opposite faces. So the faces may be painted using three colours so that faces sharing an edge are painted different colours.

PROBLEM OF THE WEEK 3

An angle in a diagram is $2\frac{1}{2}^\circ$.

How large will this angle appear to be if you look at the diagram through a stack of five magnifying glasses, each one of which magnifies by a factor of 2?

Intermediate Mathematical Challenge, 1998.

United Kingdom Mathematics Trust

PROBLEM OF THE WEEK 2

Each day throughout July I picked 300g of raspberries from my garden.

What was the total weight, in kilograms, of the raspberries that I picked that month?

Junior Mathematical Challenge 1997.

United Kingdom Mathematics Trust

Problem Of The Week 4

A ball is dropped onto a hard surface. Each time it bounces, it rebounds to exactly one third of the height from which it fell.
After the second bounce the ball rises to a height of 9 cm.

From what height was it orginally dropped?

Junior Mathematical Challenge, 1997.

United Kingdom Mathematics Trust

POTW 4 Solution

81 cm

The ball rises to a height of 9 cm after the second bounce. Therefore before the second bounce the ball fell from a height of $3 \times 9\,\text{cm} = 27\,\text{cm}$.

Thus the ball rises to a height of 27 cm after the first bounce. Therefore it was originally dropped from a height of $3 \times 27\,\text{cm} = 81\,\text{cm}$.

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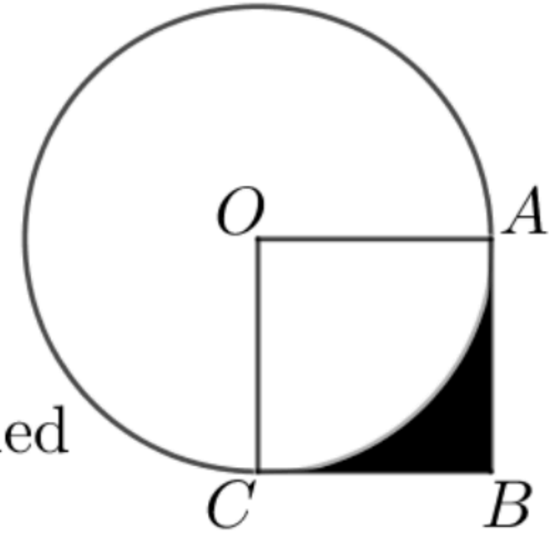
Problem Of The Week 5

The point O is the centre of a circle of radius 1 cm. OA and OC are radii of the circle. $OABC$ is a square.



What is the area of the shaded region?

Intermediate Mathematical Challenge, 1997.

United Kingdom Mathematics Trust

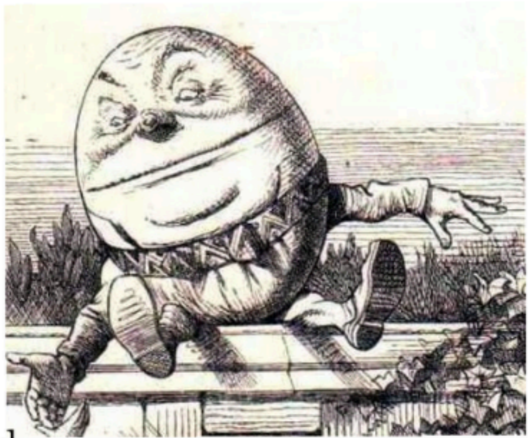
POTW 5 Solution

$\left(1 - \frac{1}{4}\pi\right)\,\text{cm}^2$ (approximately $0.215\,\text{cm}^2$)

The area of the square $OABC$ is $1\,\text{cm}^2$.
Using the formula πr^2 for the area of a circle with radius r, we see that the circle which has has radius $1\,\text{cm}$, has area $\pi(1^2)\,\text{cm}^2$, that is, $\pi\,\text{cm}^2$.
One quarter of the circle is inside the square, so this part of the circle has area $\frac{1}{4}\pi\,\text{cm}^2$.
The shaded region is the part of the square that is not inside the circle. So its area is $\left(1 - \frac{1}{4}\pi\right)\,\text{cm}^2$.

Problem Of The Week 6

Humpty Dumpty sat on a wall, admiring his new digital watch which displayed hours and minutes only. He noticed that it was 15:21 when Jack and Jill set off up the hill, but that when they later came tumbling down again his watch showed only 10:51 when they returned to the bottom of the hill. At that point Humpty realised that he'd had his watch on upside down all the time!



How long did Jack and Jill take to go up the hill and down again?

Junior Mathematical Challenge, 1997.

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POTW 6 Solution

2 hours and 10 minutes

Humpty's watch was upside down. So when he thought the time was 15:21, it was really 12:51, and when he thought it was 10:51, the actual time was 15:01.

Since Jack and Jill's time on the hill lasted from 12:51 to 15:01, it took them 2 hours and 10 minutes to go up the hill and tumble back down.

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Problem Of The Week 7

A newspaper has thirty-six pages.

Which other three pages are on the same sheet as page 10?

Junior Mathematical Challenge, 1997.

POTW 7 Solution

9, 27 and 28

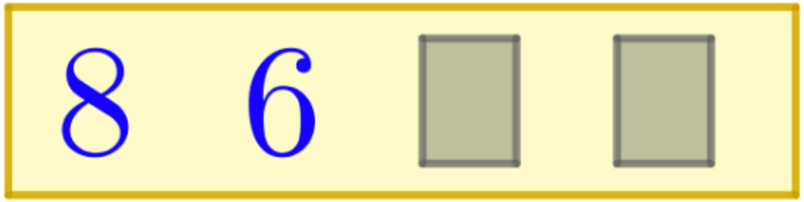
The paper has 36 pages and therefore is made up of 9 sheets each containing 4 pages.

The first sheet contains the first two and the last two pages, that is, pages 1, 2, 35 and 36.

The second sheet contains pages 3, 4, 33 and 34, the third sheet contains pages 5, 6, 31 and 32, and so on.

It follows that the fifth sheet contains pages 9, 10, 27 and 28.

Problem Of The Week 8



A four-digit number was written on a piece of paper. The last two digits were then blotted out as shown.

The complete four-digit number is divisible by three, by four and by five.

Which are the two digits that were blotted out?

Junior Mathematical Challenge, 1997.

POTW 8 Solution

4 0

The four-digit number is divisible by 4 and by 5 and so it is divisible by 10. Hence its final digit is 0.

Therefore the number is $86d0$, where d is a yet unknown digit.

The test for a number to be divisible by 3 is that the sum of its digits is divisible by 3. Since $8 + 6 + 0 = 14$, the only possibilities for d are 1, 4 and 7 making the number 8610, 8640 or 8670. Of these, only 8640 is divisible by 4. Therefore the blotted out digits are 4 0.

Problem Of The Week 9

Clearing up after the party I found two pop bottles which were full, two which were one-third full, two which were half full, two which were one-third empty, two which were half empty, and two which were completely empty.

How many bottles did I find altogether?

Junior Mathematical Challenge, 1998.

United Kingdom Mathematics Trust

POTW 9 Solution

10

The two half full bottles were the same as the two half empty bottles, but the two full bottles, the two one-third full bottles, the two one-third empty bottles, and the two completely empty bottles were all different. So the total number of bottles was $2 + 2 + 2 + 2 + 2 = 10$.

Problem Of The Week 10

In 1967 Mike McNamara cycled 445.0 km in 12 hours – a new British men's record at that time.

In the same time trial Beryl Burton completed 446.2 km.

How much faster was Beryl Burton's average speed, in metres per hour, than Mike McNamara's?

Intermediate Mathematical Challenge, 1997.

Beryl Burton (1937-1996)

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POTW 10 Solution

100

In the 12 hours Beryl Burton cycled $(446.2 - 445)\,\text{km} = 1.2$ km further than Mike McNamara. Now 1.2 km is 1200 metres and 1200 metres in 12 hours is equivalent to 100 metres per hour.

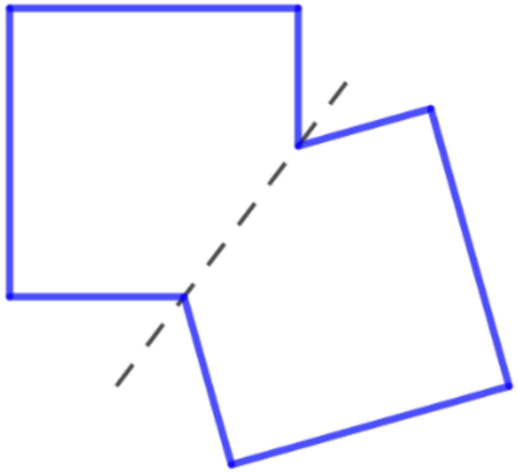
Problem Of The Week 11

A square lies on a horizontal table.

Where should an upright mirror be placed so that the part of the square on one side of the mirror and its reflection in the mirror form an octagon?

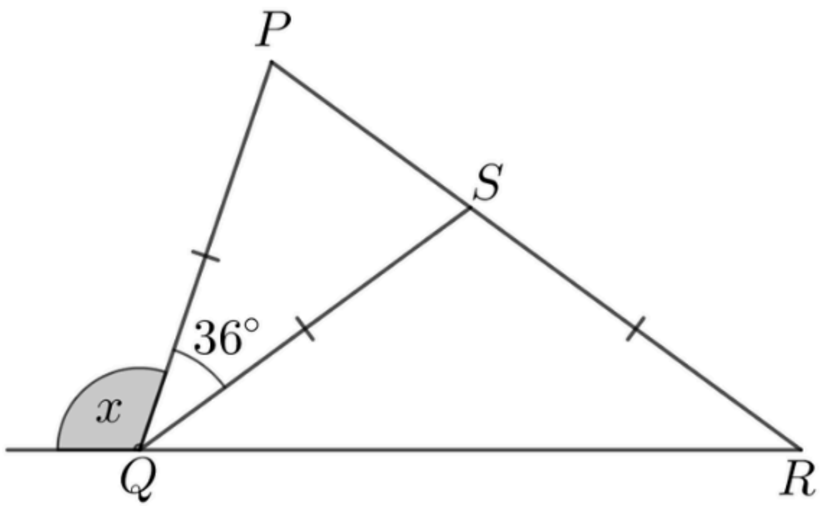
Junior Mathematical Challenge, 1998.

POTW 11 Solution



The mirror should be placed along any line that goes through two adjacent edges of the square, but not through any vertex. One example is shown above.

Problem Of The Week 12



In the triangle PQR

$$PQ = SQ = SR \quad \text{and} \quad \angle PQS = 36^\circ.$$

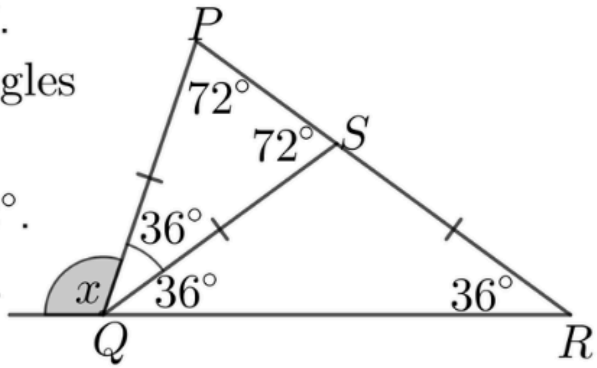
What is the size of the angle marked x?

Intermediate Mathematical Challenge, 1999.

POTW 12 Solution

$108°$

Because $PQ = SQ$, $\angle QPS = \angle QSP$.
Therefore, because the sum of the angles of the triangle PQS is $180°$,
$\angle QPS = \angle QSP = \frac{1}{2}(180 - 36)° = 72°$.
Because $SQ = SR$, $\angle SQR = \angle SRQ$.
By the External Angle Theorem,



(this theorem says that the external angle of a triangle equals the sum of the two opposite angles), $\angle PSQ = \angle SQR + \angle SRQ$.
Therefore $\angle SQR = \angle SRQ = 36°$.
Finally, by another application of the External Angle Theorem, the angle marked x is equal to $\angle QPR + \angle QRP = 36° + 72° = 108°$.

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Problem Of The Week 13

At the first ever *World Worm-Charming Championship* held at Wollaston, Cheshire in 1980, Tom Shufflebottom charmed a record number of 510 worms out of his $3\,\text{m} \times 3\,\text{m}$ patch of ground in 30 minutes.

If the worms, of average length 20 cm, stopped wriggling and were laid out end to end round his patch, approximately how many times round it would they have stretched?

Junior Mathematical Challenge, 1998.

United Kingdom Mathematics Trust

POTW 13 Solution

$8\frac{1}{2}$

The total length of 510 worms, each of length 20 cm, is (510×20) cm $= 10\,200$ cm $= 102$ m.

The perimeter of a 3 m $\times$ 3 m patch of ground is $(3 + 3 + 3 + 3)$ m $= 12$ m.

Therefore the number of times round this patch that the worms would have stretched is $\dfrac{102}{12} = 8\frac{1}{2}$.

Problem Of The Week 14

Inspector Remorse estimates that he can solve the average murder in 50 hours, a bank robbery in half that time, and a car theft in one third of the time he takes to solve a bank robbery.

How many hours would he expect to take in solving two murders, six car thefts and four bank robberies?

Intermediate Mathematical Challenge, 1999.

United Kingdom Mathematics Trust

POTW 14 Solution

250

Inspector Remorse expects to take 50 hours to solve a murder. He expects to solve a bank robbery in half that time, that is, in $\frac{1}{2}(50)$ hours. He expects to solve a car theft in one third of this latter time, that is, in $\frac{1}{3}(\frac{1}{2}(50)) = \frac{1}{6}(50)$ hours.

So, the number of hours in which he expects to solve two murders, six car thefts and four bank robberies is $2 \times 50 + 6 \times \frac{1}{6}(50) + 4 \times \frac{1}{2}(50) = 100 + 50 + 100 = 250$.

Problem Of The Week 15

On her birthday a week ago Granny said, "Today I am 84 years old - not counting my Sundays".

How old is she really today?

Junior Mathematical Challenge, 1999.

United Kingdom Mathematics Trust

POTW 15 Solution

98

Granny counts only 6 days in each week.

So 84 is only six-sevenths of her true age.

Therefore her age is $\dfrac{7}{6} \times 84 = 7 \times 14 = 98$.

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Problem Of The Week 16

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{n}\right)$$

For which positive integers n is the product

$$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{n}\right)$$

exactly equal to an integer?

Intermediate Mathematical Challenge, 1999.

POTW 16 Solution

n has to be odd

$$
\begin{aligned}
\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots\left(1+\frac{1}{n}\right)
&= \frac{\cancel{3}}{2}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{5}}{\cancel{4}}\times\ldots\times\frac{n+1}{\cancel{n}} \\
&= \frac{n+1}{2},
\end{aligned}
$$

after a lot of cancelling.

For $\dfrac{n+1}{2}$ to be an integer, $n+1$ needs to be even and therefore n has to be odd.

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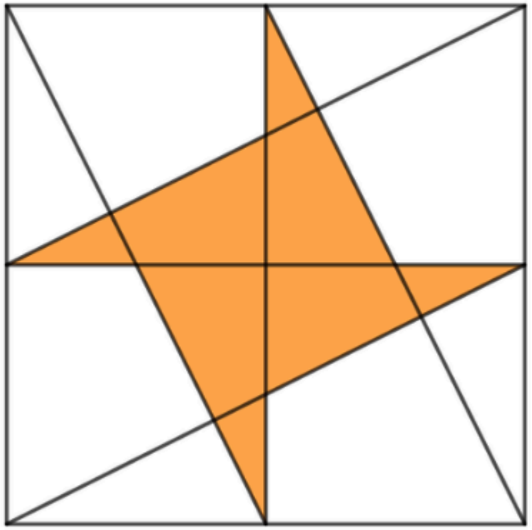
Problem Of The Week 17



In the diagram, a corner of the orange star is at the midpoint of each side of the large square.

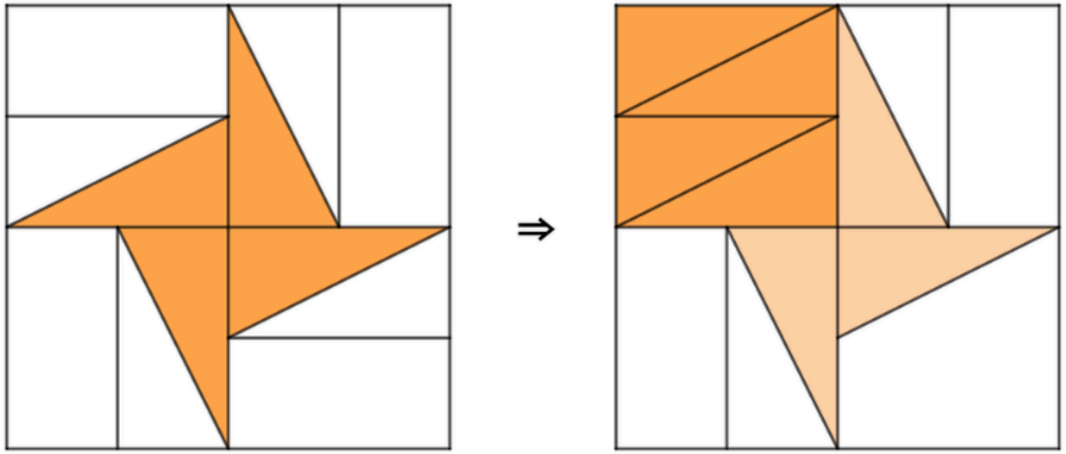
What fraction of the large square is coloured orange?

Junior Mathematical Challenge 2001

United Kingdom Mathematics Trust

POTW 17 Solution

$\frac{1}{4}$



It may be seen that the star is made up of four right-angled triangles, each of which covers one quarter of one quarter of the large square.
Therefore together they cover one quarter of the large square

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Problem Of The Week 18

My bus fare is 44p. The driver can give me change. What is the smallest number of coins which must change hands when I pay this fare?

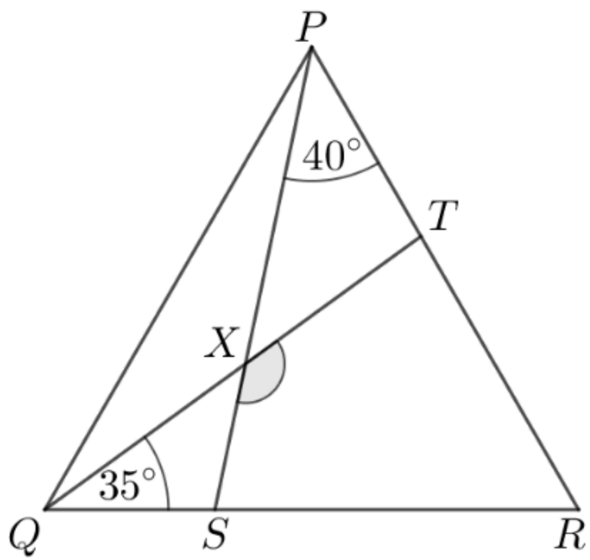
Junior Mathematical Challenge 2001

POTW 18 Solution

3

I can pay the 44p fare by giving the driver a 50p coin, and receiving a 5p coin and a 1p coin in change.
In this way 3 coins change hands. It may be checked that payment of 44p cannot be achieved if only 2 coins change hands.

Problem Of The Week 19



The triangle PQR is equilateral.
$\angle SPR = 40^\circ$. $\angle TQR = 35^\circ$.

What is size of the marked angle $\angle SXT$?

Junior Mathematical Challenge 2001

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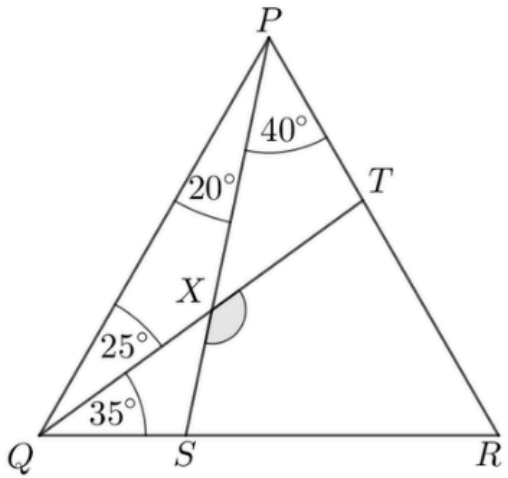
POTW 19 Solution

135°



The triangle PQR is equilateral. Therefore $\angle QPR = \angle PQR = 60^\circ$.
Therefore $\angle QPX = \angle QPS = \angle QPR - \angle SPR = 60^\circ - 40^\circ = 20^\circ$,
and $\angle PQX = \angle PQT = \angle PQR - \angle TQR = 60^\circ - 35^\circ = 25^\circ$.
The sum of the angles in the triangle PXQ is 180°.
Therefore $\angle PXQ = 180^\circ - 20^\circ - 25^\circ = 135^\circ$.
Vertically opposite angles are equal.
Therefore $\angle SXT = \angle PXQ = 135^\circ$.

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Problem Of The Week 20

18			
13	15		
	10	11	17
	x	16	14

In a magic square, each row, each column and each of the two main diagonals have the same total.

In this partially completed magic square, which number should replace x?

Intermediate Mathematical Challenge 2000

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POTW 20 Solution

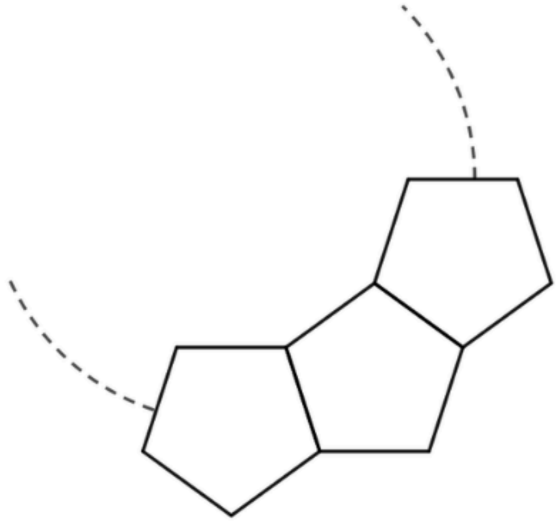
21

The numbers on the diagonal have total $18 + 15 + 11 + 14 = 58$. Therefore 58 is the total of each row and column. Hence the green number is $58 - (10 + 11 + 17) = 20$. Hence the orange number is $58 - (18 + 13 + 20) = 7$. We deduce that the red number is $58 - (7 + 16 + 14) = 21$.

18			
13	15		
20	10	11	17
7	21	16	14

Can you complete the magic square?

Problem Of The Week 21



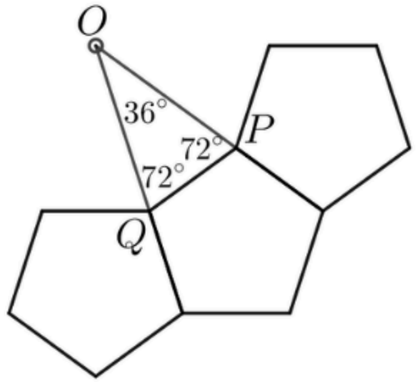
Regular pentagons are placed together to form a ring in the manner shown. The diagram shows the first three pentagons.

How many *more* are needed to complete the ring?

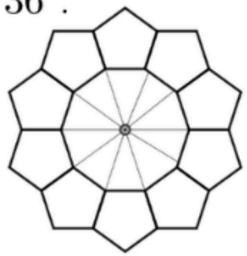
Intermediate Mathematical Challenge 2001

POTW 21 Solution

7



The exterior angle of a regular pentagon is $72°$.
Therefore $\angle QPO = \angle PQO = 72°$.
Hence $\angle QOP = 180° - 72° - 72° = 36°$.



Because $360 \div 36 = 10$, it follows that 10 pentagons make a complete ring. Hence 7 more pentagons are needed to complete the ring.

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Problem Of The Week 22

A long-sleeved shirt has 8 front buttons and 2 cuff buttons. A short-sleeved shirt has 6 front buttons and no cuff buttons.
The factory that makes *Slimboy Shirts* uses 10 times as many front buttons as cuff buttons.

What is the ratio of long-sleeve shirts to short-sleeve shirts produced by the factory?

Intermediate Mathematical Challenge 2001

POTW 22 Solution

$1:2$

Suppose that the factory makes x long-sleeved shirts and y short-sleeved shirts.
Then it uses $8x + 6y$ front buttons and $2x$ cuff buttons.
Because the factory uses 10 times as many front buttons as cuff buttons $8x + 6y = 10 \times 2x$. Therefore $8x + 6y = 20x$. Hence $6y = 12x$, and so $y = 2x$.
Therefore the ratio of long-sleeved shirts to short-sleeved shirts produced by the factory is $1 : 2$.

United Kingdom Mathematics Trust

Problem Of The Week 23

Sally has 72 small wooden cubes, each measuring $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$. She arranges them all so that they form a cuboid.
The perimeter of the base of the cuboid is $16\,\text{cm}$.

What is the height of the cuboid?

Junior Mathematical Challenge 2002

POTW 23 Solution

6 cm

Suppose that the base of the cuboid measures $a\,\text{cm} \times b\,\text{cm}$. Here a and b will be integers because the cuboid is made from $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$ cubes.
The base has two edges with length $a\,\text{cm}$ and two edges with length $b\,\text{cm}$. Hence its perimeter is $2a + 2b\,\text{cm}$.
Therefore $2a + 2b = 16$. Hence $a + b = 8$.
Each layer of the cuboid contains ab small cubes.
Therefore ab is a factor of 72. The only pair of integers a, b with sum 8 whose product is factor of 72 is 2,6.
Therefore each layer contains $2 \times 6 = 12$ small cubes.
Hence there are $72 \div 12 = 6$ layers.
Therefore the height of the cuboid is $6\,\text{cm}$.

Problem Of The Week 24

The $\Pi\upsilon\theta\alpha\gamma o\rho\alpha\zeta$ Patisserie

The Pythagoras Patisserie sells triangular cakes at 39p each and square buns at 23p each.
For her party, Helen spent exactly £5.12 on an assortment of these cakes and buns.

How many items in total did she buy?

Intermediate Mathematical Challenge 2001

POTW 24 Solution

16

Let x be the number of cakes that Helen bought and let y be the number of buns that she bought.
Then $39x + 23y = 512$.
We have two unknowns but only one equation.
However x and y are non-negative integers. This enables us to find their values. We can rewrite the the equation as $23y = 512 - 39x$. Since $y \geq 0$, we have $39x \leq 512$ and hence $x \leq 13$. It also follows that $512 - 39x$ is a multiple of 23.
It may now be checked that $x = 9$ is the only value that satisfies these conditions. When $x = 9$,
$23y = 512 - 9 \times 39 = 161$. Hence $y = 161 \div 23 = 7$.
Therefore Helen bought 9 triangular cakes and 7 square buns, making a total of 16 items.

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Problem Of The Week 25

$$1\,2\,3\,4\diamond 6\,7\,8$$

The eight-digit number $1234\diamond 678$ is a multiple of 11.

Which digit is represented by $\diamond$?

Junior Mathematical Challenge, 1999.

POTW 25 Solution

9

The test for whether an integer n is divisible by 11 is as follows:
Let X be the sum of those digits of n in odd positions, (counting from the left) and let Y be the sum the digits in even positions.
Then n is divisible by 11 provided that $X = Y$ or or the difference between X and Y is a multiple of 11.
Here $X = 1 + 3 + \diamond + 7 = 11 + \diamond$ and $Y = 2 + 4 + 6 + 8 = 20$. These are equal provided that $\diamond = 9$.

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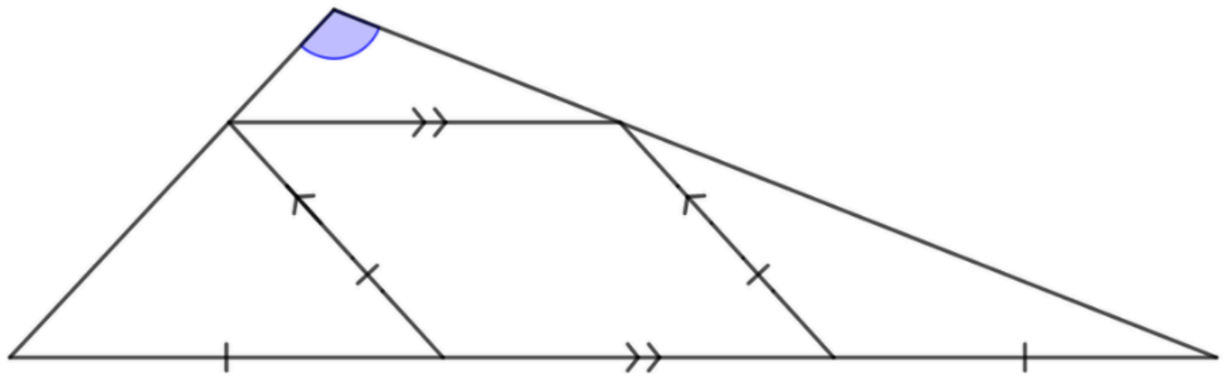
Problem Of The Week 26



The diagram, which is not drawn to scale, shows a parallelogram inside a triangle.

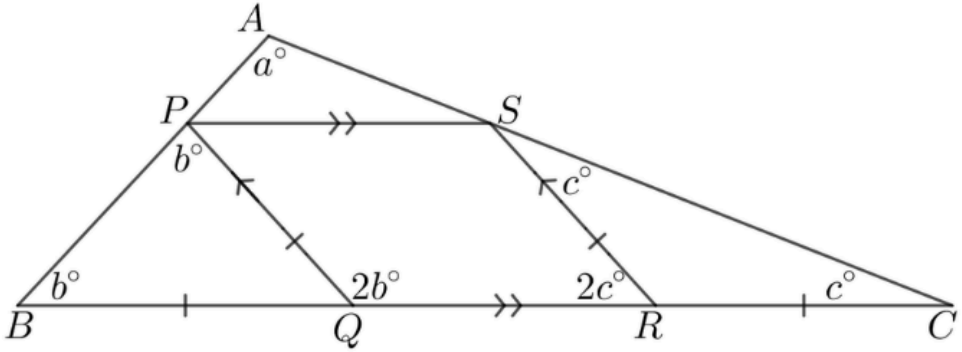
The marked line segments all have the same length.

What is the size of the marked angle?

Intermediate Mathematical Challenge, 2002.

POTW 26 Solution

90°



We label the points as shown. Let $\angle BAC = a^\circ, \angle ABC = b^\circ$ and $\angle BCA = c^\circ$.

Because $PQ = BQ$, we have $\angle BPQ = \angle PBQ = b^\circ$.

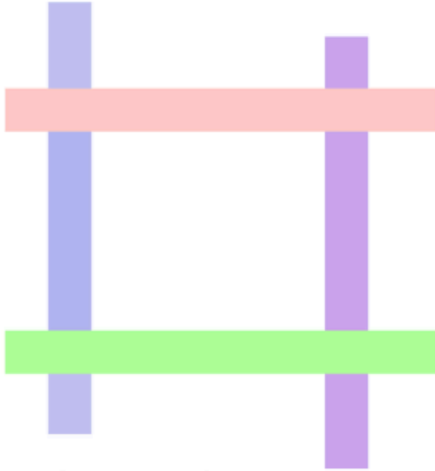
Therefore, by the External Angle Theorem,

$$\angle PQC = \angle PBQ + \angle BPQ = b^\circ + b^\circ = 2b^\circ.$$

Similarly, $\angle BRS = 2c^\circ$. Now, because $PQRS$ is a parallelogram, $2b^\circ + 2c^\circ = 180^\circ$. Therefore $b^\circ + c^\circ = 90^\circ$.

The sum of the angles of the triangle ABC is 180°. Therefore $a^\circ + b^\circ + c^\circ = 180^\circ$. Hence $a^\circ = 180^\circ - (b^\circ + c^\circ) = 180^\circ - 90^\circ = 90^\circ$.

Problem Of The Week 27



Four coloured rectangular paper strips, each measuring 10 cm by 1 cm are laid flat on a table. Each strip is at right angles to two other strips as shown

What is the area of the part of the table covered by the strips?

Junior Mathematical Challenge, 2000.

United Kingdom Mathematics Trust

POTW 27 Solution

$36\ \text{cm}^2$

Each strip measures $10\,\text{cm} \times 1\,\text{cm}$ and hence has area $10\,\text{cm}^2$. So the total area of the four strips is $40\,\text{cm}^2$. This counts the area where the strips overlap twice. Their overlaps form four $1\,\text{cm} \times 1\,\text{cm}$ squares, making a total area of $4\,\text{cm}^2$.

Therefore the area of the part of the table covered by the strips is $40\,\text{cm}^2 - 4\,\text{cm}^2 = 36\,\text{cm}^2$.

Problem Of The Week 28

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{a}=?$$

a, b, c and d are non-zero numbers.

$$a = b - c,$$
$$b = c - d,$$
and
$$c = d - a.$$

What is the value of $\dfrac{a}{b}+\dfrac{b}{c}+\dfrac{c}{d}+\dfrac{d}{a}$?

United Kingdom Mathematics Trust

POTW 28 Solution

$\frac{1}{2}$

From $a = b - c$ and $b = c - d$, it follows that

$$d = c - b = -(b - c) = -a.$$

Therefore, from $c = d - a$, we can deduce that

$$c = (-a) - a = -2a.$$

Hence $b = c - d = (-2a) - (-a) = -2a + a = -a.$

It follows that

$$\begin{aligned}\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} &= \frac{a}{-a} + \frac{-a}{-2a} + \frac{-2a}{-a} + \frac{-a}{a} \\ &= -1 + \frac{1}{2} + 2 - 1 = \frac{1}{2}.\end{aligned}$$

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Problem Of The Week 29

In a swimming match between two schools, each school enters two swimmers for each race.

In each race 5 points are awarded for first place, 3 points for second place, 2 points for third place and 1 point for fourth place

After six races, no swimmer had been disqualified. The number of points scored by the leading team uses the same two digits as the score of the team in second place, but in the reverse order.

What is the difference in their scores at this stage?

Junior Mathematical Challenge, 1998.

United Kingdom Mathematics Trust

POTW 29 Solution

18

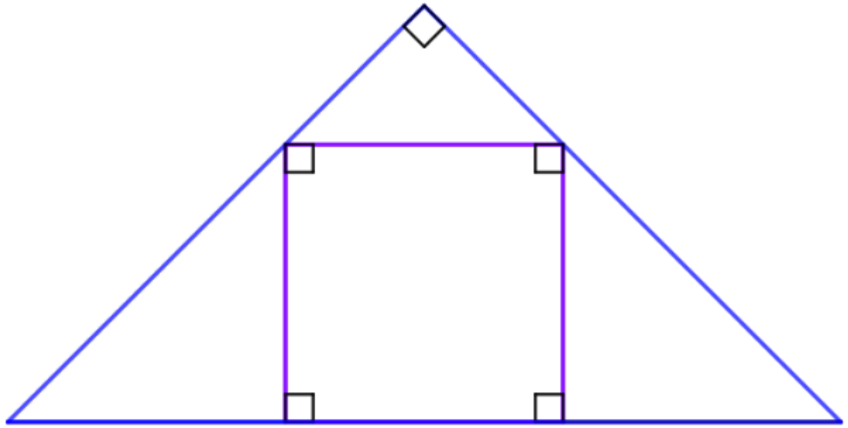
The total number of points scored in one race is $5 + 3 + 2 + 1 = 11$. Therefore, the total number of points scored after 6 races is $6 \times 11 = 66$.

We therefore seek two numbers whose digits are in reverse order and whose sum is 66.

The only possibilities are 42 and 24 or 51 and 15. However a team can score at most 8 points in each race and so cannot score 51 after 6 races. Therefore the leading team must have 42 points, and the second team 24 points. Note that this could be achieved, for example, by the leading team coming first and third in each race.

$42 - 24 = 18$.

Problem Of The Week 30



The diagram shows a square inscribed in a right-angled isosceles triangle.

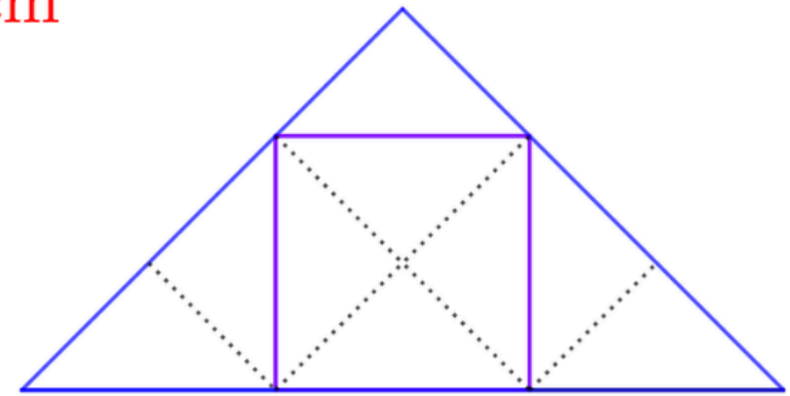
The area of the triangle is $36\,\text{cm}^2$.

What is the area of the square?

Intermediate Mathematical Challenge, 2001.

POTW 30 Solution

$16\,\text{cm}^2$

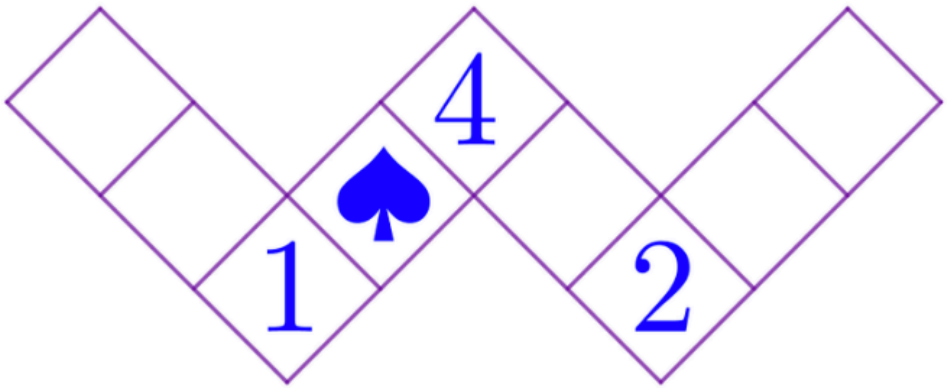


In the diagram we have divided the triangle into 9 congruent smaller triangles. The square is made up of 4 of these triangles.

The large triangle has area $36\,\text{cm}^2$.

Therefore the area of square is $\frac{4}{9} \times 36\,\text{cm}^2 = 16\,\text{cm}^2$.

Problem Of The Week 31



The integers from 1 to 9, inclusive, are to be put, one number to a square, in the figure shown, so that the total of the numbers in each of the four lines is the same.

Which integer should replace ♠ ?

Junior Mathematical Challenge, 2001.

POTW 31 Solution

8

Let S be the total of the numbers in each of the four lines.

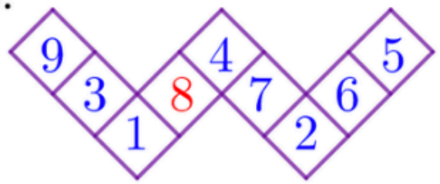
Then $4S$ is the sum of these totals and is therefore the sum of all the integers from 1 to 9, with the numbers 1, 2 and 4 counted twice, as they each occur in two of the lines. Therefore

$4S = (1+2+3+4+5+6+7+8+9)+(1+2+4)$

$= 45 + 7 = 52$. Hence $S = 13$.

It follows that 8 should replace ♠.

(The diagram shows one way to obtain a total of 13 in each line.)



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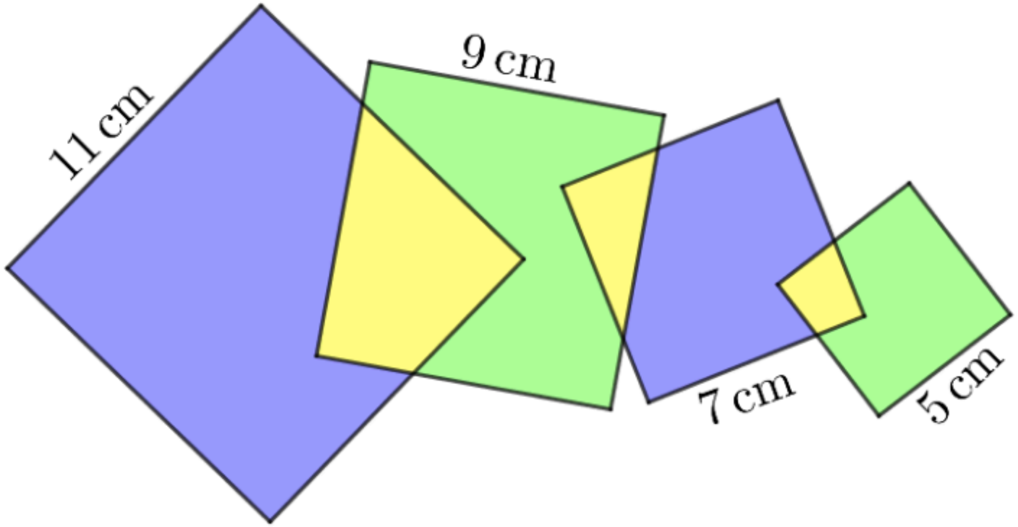
Problem Of The Week 32



The diagram shows four overlapping squares with sides of length 11 cm, 9 cm, 7 cm and 5 cm.

What is the difference between the total area shaded blue and the total area shaded green?

Junior Mathematical Challenge, 2001.

UKMT

United Kingdom Mathematics Trust

POTW 32 Solution

$64\,\text{cm}^2$

The area that is blue or yellow is the area of the squares with sides of length $11\,\text{cm}$ and $7\,\text{cm}$.
Therefore this area is
$11^2\,\text{cm}^2 + 7^2\,\text{cm}^2 = 121\,\text{cm}^2 + 49\,\text{cm}^2 = 170\,\text{cm}^2$.
Similarly the area that is green or yellow is
$9^2\,\text{cm}^2 + 5^2\,\text{cm}^2 = 81\,\text{cm}^2 + 25\,\text{cm}^2 = 106\,\text{cm}^2$.
The difference between these areas is the difference between the area shaded blue and the area shaded green. Hence this area is $170\,\text{cm}^2 - 106\,\text{cm}^2 = 64\,\text{cm}^2$.

Problem Of The Week 33

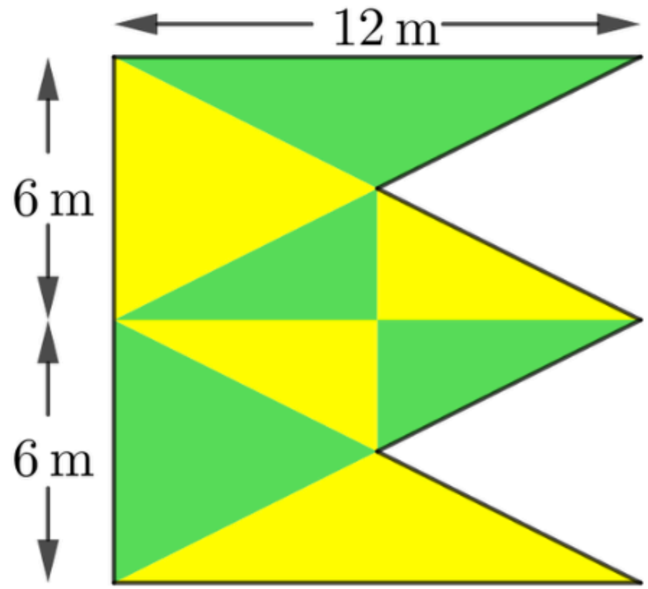
Gill has recently moved to a new house, which has a three-digit number.

The sum of this number and its three individual digits is 429.

What is the *product* of the three digits that make up the house number?

Junior Mathematical Challenge, 2003

Problem Of The Week 34



What is the area of the part of this pennant that is coloured green?

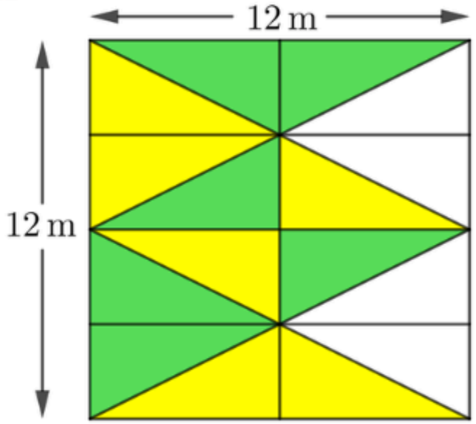
Intermediate Mathematical Challenge, 2002.

UKMT

United Kingdom Mathematics Trust

POTW 34 Solution

$54\,\mathrm{m}^2$



The diagram shows that the pennant is part of a square measuring $12\,\mathrm{m} \times 12\,\mathrm{m}$ and hence with area $12^2\,\mathrm{m}^2{=}144\,\mathrm{m}^2$.

The square is divided into 16 congruent triangles.
The green area is made up of 6 of these triangles.
Therefore the green area is

$$\frac{6}{16} \times 144\,\mathrm{m}^2 = 54\,\mathrm{m}^2.$$

Problem Of The Week 35

Lisa's bucket does not have a hole in it and weighs 21 kg when full of water. After she pours out half the water from bucket, it weights 12 kg.

What is the weight of the empty bucket?

Junior Mathematical Challenge, 2002.

UKMT

United Kingdom Mathematics Trust

POTW 35 Solution

$3\,\text{kg}$

The full bucket weights $21\,\text{kg}$. The half-full bucket weighs $12\,\text{kg}$. Therefore the weight of water that half fills the bucket is
$21\,\text{kg} - 12\,\text{kg} = 9\,\text{kg}$.
Hence the weight of water that fills the bucket is $2 \times 9\,\text{kg} = 18\,\text{kg}$.
Therefore the weight of the empty bucket is
$21\,\text{kg} - 18\,\text{kg} = 3\,\text{kg}$.

Problem Of The Week 36

The Pythagoras School of Music has 100 students. Of these, 60 are in the band and 20 are in the orchestra. There are 12 students who are in both the band and the orchestra

How many of the students are in neither the band nor the orchestra?

Junior Mathematical Challenge, 2002.

UKMT

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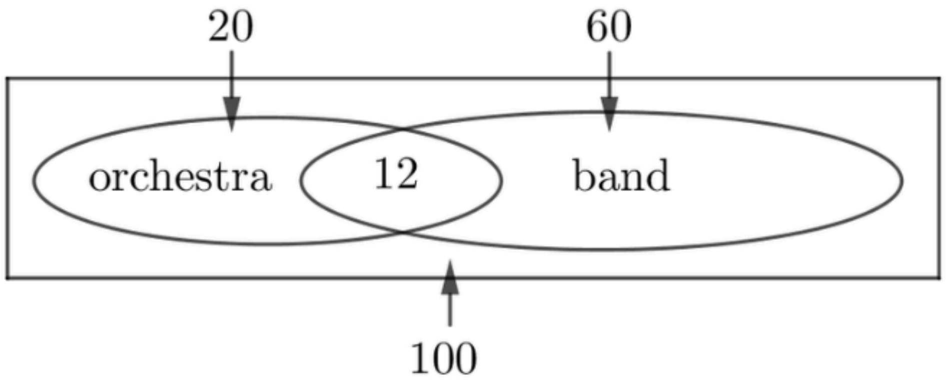
POTW 36 Solution

32



When we add the 60 students in the band to the 20 students in the orchestra, we count the 12 students who are in both twice. Therefore the number of the students in the band or the orchestra or both is $60 + 20 - 12 = 68$.

There are 100 students in the School. Therefore the number of students in neither the band nor the orchestra is $100 - 68 = 32$.

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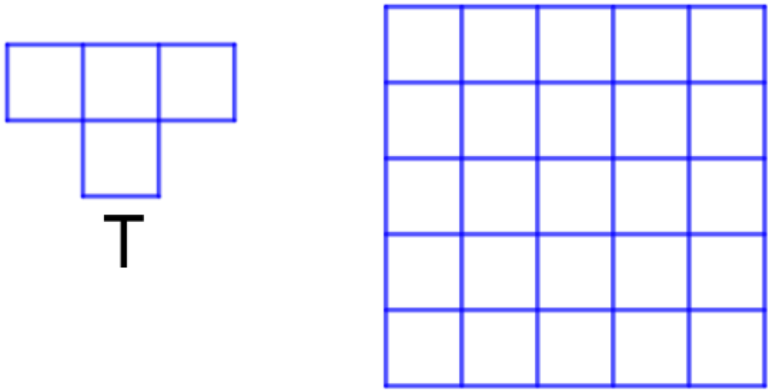
Problem Of The Week 37



What is the maximum number of pieces with the shape T which can be placed in the 5×5 grid shown without overlapping, and with their edges along the lines of the grid?

Intermediate Mathematical Challenge, 2003.

UKMT
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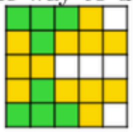
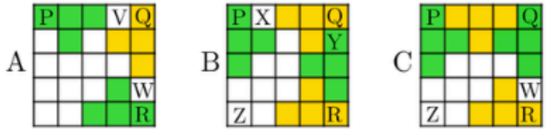
POTW 37 Solution

5

It is easy to fit five T pieces in the grid. One way to do this is shown in the diagram on the right. To show that the maximum is five, we need to show that it is not possible to fit six T pieces in the grid.





If six T pieces are fitted in the grid, they cover 24 squares of the grid, leaving just one square uncovered. Therefore at least three of the corner squares are covered. Let us assume that the squares marked P, Q and R in the diagrams A, B and C are all covered.
These diagrams show the only three ways in which the squares P and Q can be covered by T pieces. In case A, the square R can only be covered as shown, leaving V and W uncovered. In case B, Y can only be covered as shown, and so R must be covered as shown. But then X and Z cannot be covered. In case C, R can only be covered as shown, but then W and Z cannot be covered. So six T pieces cannot be fitted in the grid.
Hence five is the maximum.

Problem Of The Week 38

The White Rabbit has an appointment to see the Red Queen at 4pm every day apart from weekends. On Monday he arrives 16 minutes late. Each day after after that he hurries more and more and so manages to halve the amount of time that he arrives late each day.

On what day of the week does he arrive just 15 seconds late?

Junior Mathematical Challenge, 2004.

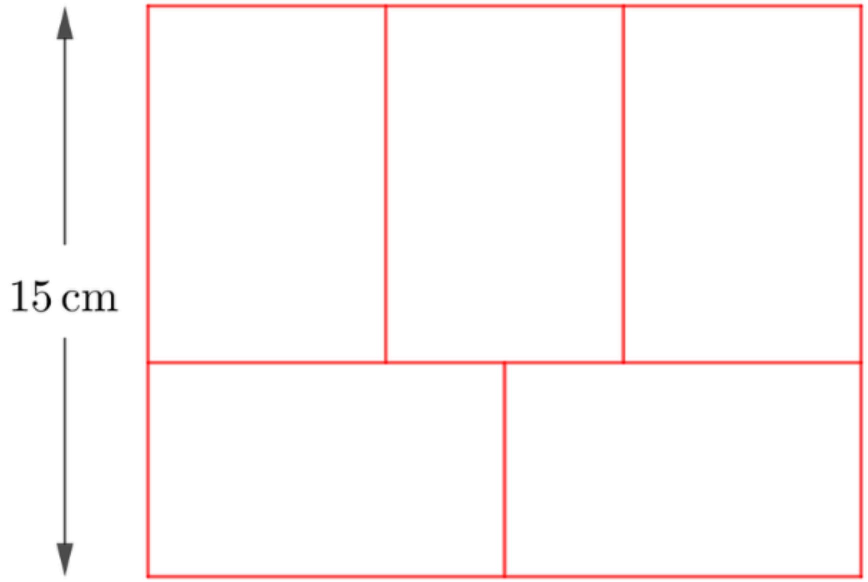
POTW 38 Solution

Tuesday

The White Rabbit halves the amount of time by which he is late each weekday. We can therefore set out these times in the following table:

day of week	time late
Monday	16 minutes
Tuesday	8 minutes
Wednesday	4 minutes
Thursday	2 minutes
Friday	1 minute
:	
Monday	30 seconds
Tuesday	15 seconds

Problem Of The Week 39



Five identical rectangles fit together, as shown in the diagram which is not drawn to scale.

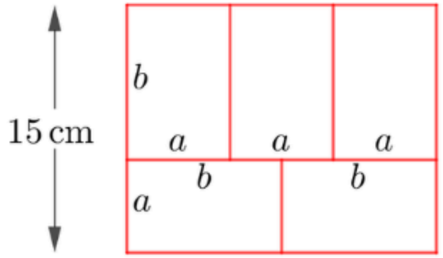
What is the total area that they cover?

Junior Mathematical Challenge, 2004.

United Kingdom Mathematics Trust

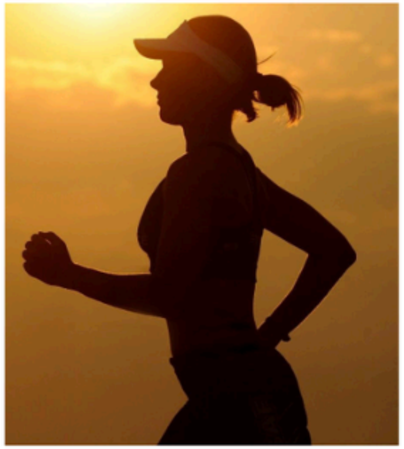
POTW 39 Solution

$270\,\text{cm}^2$



We suppose that each rectangle has width a cm and length b cm. Thus the five identical rectangles make up an area of $15\,\text{cm} \times 2b\,\text{cm}$.
We see from the diagram that $a + b = 15$ and $3a = 2b$. It follows from the second equation that $a = \frac{2}{3}b$. Hence, by the first equation $\frac{2}{3}b + b = 15$. It follows that $\frac{5}{3}b = 15$. Therefore $b = \frac{3}{5} \times 15 = 9$. Hence $2b = 18$. Therefore the total area covered by the rectangles is $15\,\text{cm} \times 18\,\text{cm} = 270\,\text{cm}^2$.

Problem Of The Week 40



After a year's training, Minny Midriffe increased her average speed in the London Marathon by 25%.

By what percentage did her time decrease?

Intermediate Mathematical Challenge, 2003.

United Kingdom Mathematics Trust

POTW 40 Solution

20%

When Minni Midriffe's speed is increased by 25%, her speed is multiplied by $1\frac{1}{4}$, that is, by $\frac{5}{4}$.

Therefore her previous time is multiplied by $\frac{4}{5}$.

So her new time is 80% of her previous time.

Hence her time is decreased by 20%.